# MTH 301: Group Theory

## Assignment IV: Alternating groups, Composition Series, Derived Series, and Solvable Groups

## Practice assignment

1. Show that there exists no proper subgroup of $A_4$ generated by an element of order 2 and an element of order 3.

2. Show that for $n \geq 5$, $S_n$ has no proper normal subgroups besides $A_n$.

3. Show that for $n \geq 3$, $A_n$ contains a subgroup isomorphic to $S_{n-2}$.

4. Prove that no subgroup of $S_4$ is isomorphic to $Q_8$.

5. If $\sigma$ and $\tau$ are 3-cycles in $S_n$, when show that $\langle \sigma, \tau \rangle$ is isomorphic to $\mathbb{Z}_3$, $A_4$, $A_5$, or $\mathbb{Z}_3 \times \mathbb{Z}_3$.

6. Show that $A_4$ is solvable.

7. Establish the assertion in 7.1 (iii) of the Lesson Plan.

8. Describe all composition series' for $D_8$ and $Q_8$.

9. Show that the quotient group of a solvable group is solvable.

10. Let $G$ be a solvable groups and $H \trianglelefteq G$ be a non-trivial subgroup. Show that there exists a non-trivial abelian subgroup $A$ of $H$ such that $A \trianglelefteq G$.

11. Without using the Feit-Thompson Theorem, show that the following statements are equivalent.

    (a) Every group of odd order is solvable.

    (b) Then only simple groups of odd order are of prime order.

## Problems for submission

(Due 03/11/2023)

- Solve problems 2, 7, 9, and 11 from the practice problems above.